Quantifying Uncertainty in Climate Reanalyses

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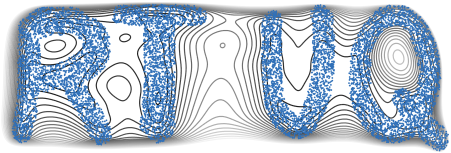
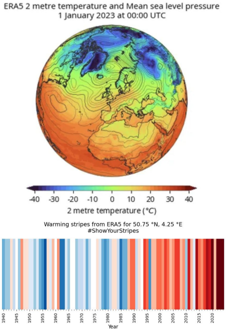


Context of the climate reanalyses

What are climate reanalyses?

- ERA, NCEP, GLORYS, etc.
- state of the atmosphere
- or state of the ocean
- over a long period of time
- used in a lot of studies



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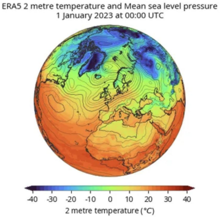

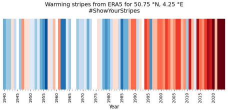

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- probabilistic approach (data assimilation)
- output the ensemble mean + ensemble spread

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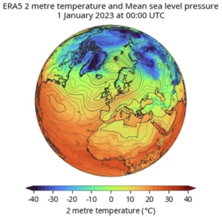

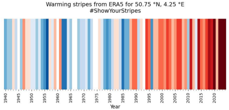

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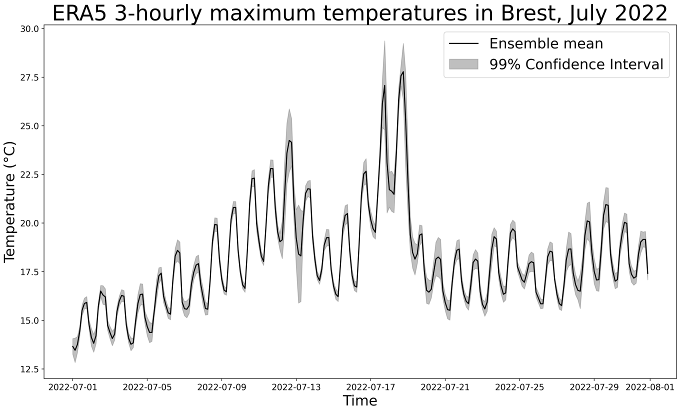
⇒ **Are climate reanalyses accurate (mean) and reliable (spread)?**

Example of ERA5 in Brest

Is ERA5 reliable?

- ground temperature in Brest, France
- during strong heat wave in 2022
- 39.3 degrees on July 18



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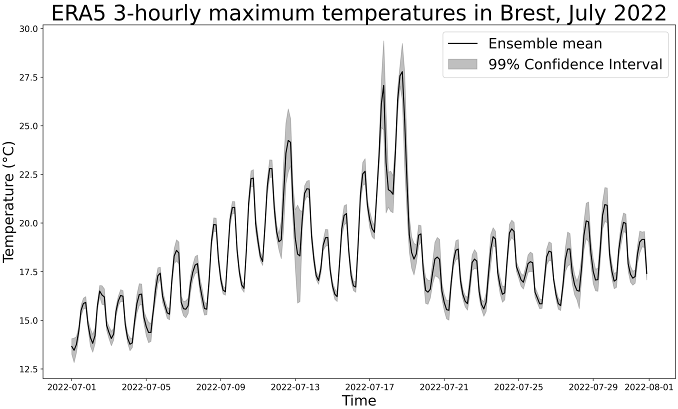
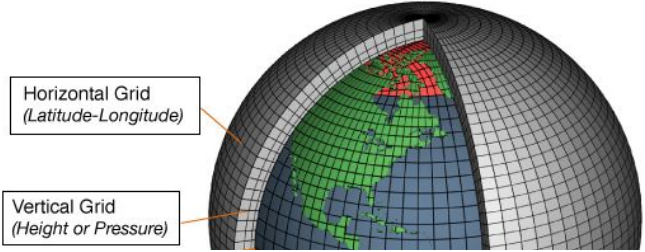


$\Rightarrow$ **39.3 is not included (at all) in the 99% confidence interval**

Data assimilation formulation

Observation, model, and state:

- we use observations **y** (partial, noisy)
- we use a model $\mathcal{M}$ (general circulation model)
- we want to estimate the state **x** (everywhere, anytime)



Grid of a general circulation model in meteorology
State x = [U, V, T, P, H] at each grid point

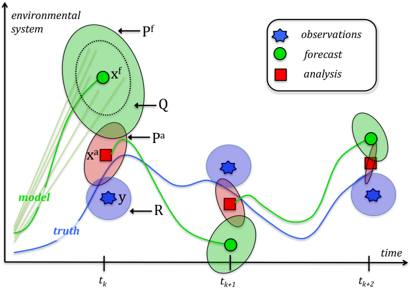
Data assimilation formulation

State-space model:

$$\mathbf{x}(t+dt) = \mathcal{M}(\mathbf{x}(t)) + \boldsymbol{\eta}(t)$$
$$\mathbf{y}(t) = \mathbf{H}\mathbf{x}(t) + \boldsymbol{\epsilon}(t)$$

Error terms:

$$\boldsymbol{\eta}(t) \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}(t))$$
$$\boldsymbol{\epsilon}(t) \sim \mathcal{N}(\mathbf{0}, \mathbf{R}(t))$$



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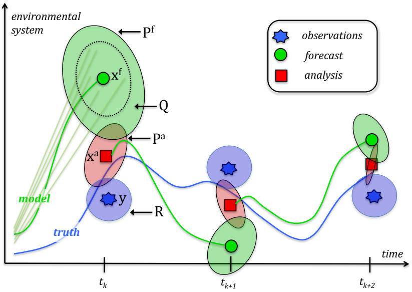

⇒ **Calibration of Q and R covariance matrices is a key point**[1]

[1]Dee, D. P. (1995). On-line Estimation of Error Covariance Parameters for Atmospheric Data Assimilation. *Monthly Weather Review*

Illustrative example

Equations:

$$x(t+1) = 0.95x(t) + \eta(t)$$
$$y(t) = x(t) + \epsilon(t)$$

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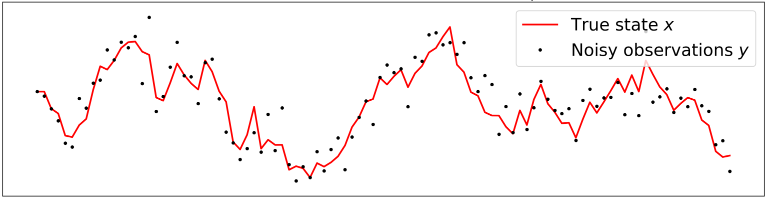
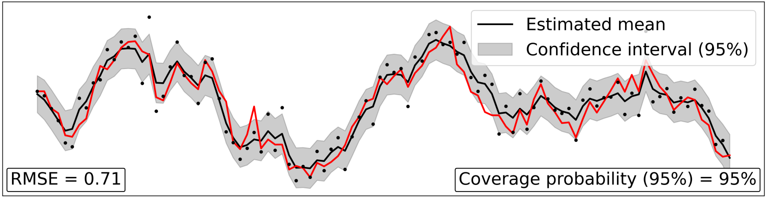
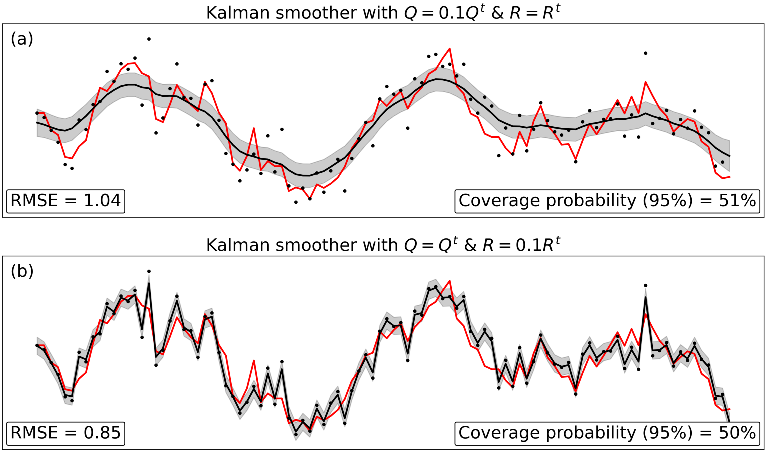
Simulated data from a linear Gaussian state-space model



Kalman smoother with $Q = Q^t$ & $R = R^t$



Bad Q/R ratio (under-calibration of Q or R)



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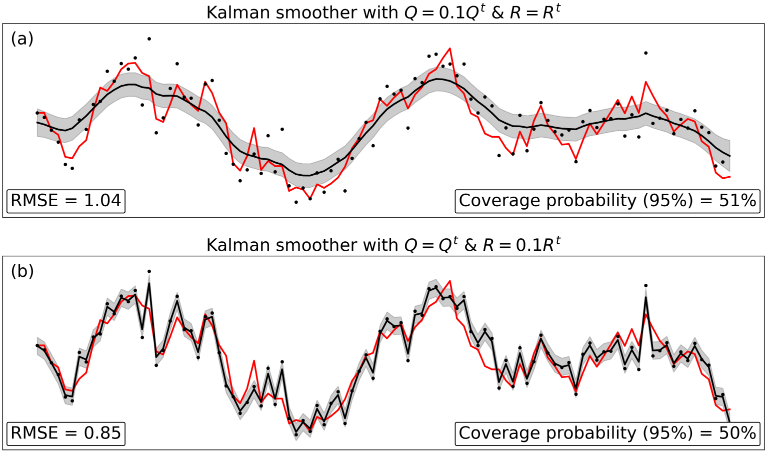

Bad Q/R ratio:

- (a) $Q/R = 0.1 \Rightarrow$ **Too confident in the model**
- (b) $Q/R = 10. \Rightarrow$ **Too confident in the observations**

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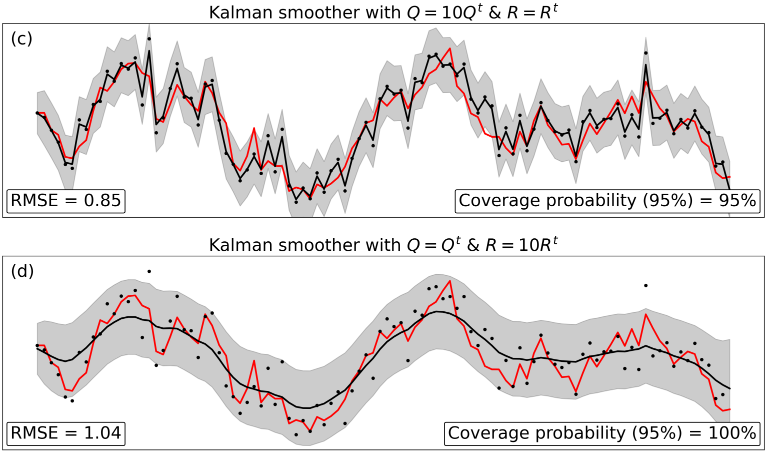

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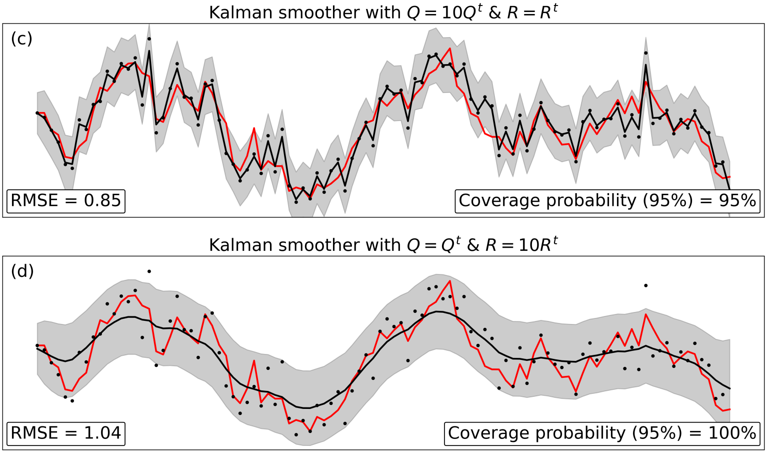
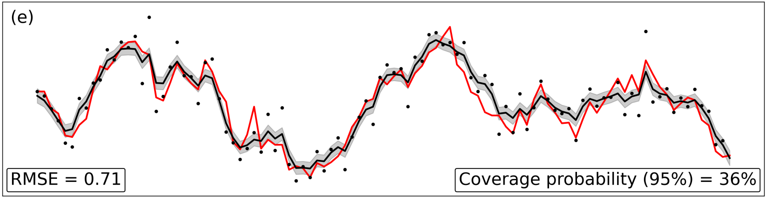


Bad Q/R ratio:

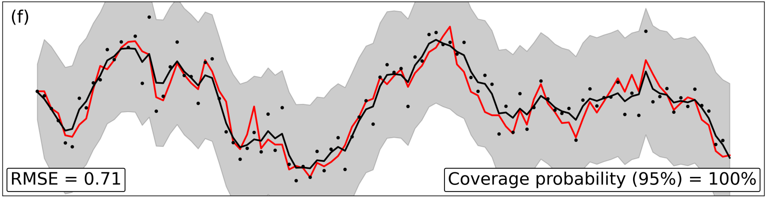
- (c) $Q/R = 10.$ $\Rightarrow$ **Same RMSE as figure (b)**
- (d) $Q/R = 0.1$ $\Rightarrow$ **Same RMSE as figure (a)**

Good Q/R ratio

Kalman smoother with $Q = 0.1Q^t$ & $R = 0.1R^t$



Kalman smoother with $Q = 10Q^t$ & $R = 10R^t$



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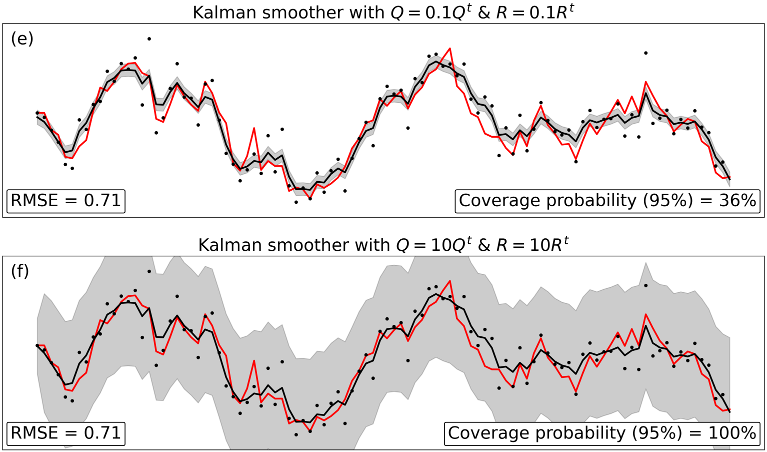


Good Q/R ratio:

- (e) Q and R too low ⇒ **Overconfident in the results**
- (f) Q and R too high ⇒ **Underconfident in the results**

Estimation of both **Q** and **R**

State-of-the-art paper[1]:

- review methods from different communities
- clear mathematical formulation and comparison

[1]Tandeo et al. (2020). A Review of Innovation-based Methods to Jointly Estimate Model and Observation Error Covariance Matrices in Ensemble Data Assimilation. *Monthly Weather Review*

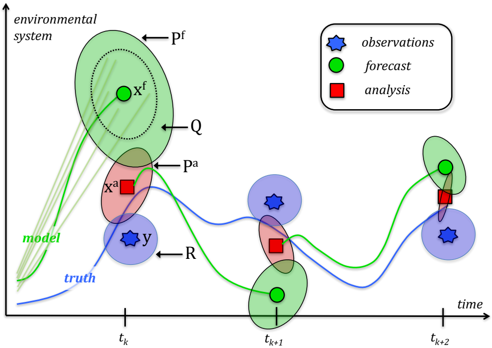
Maximization of the innovation likelihood

Definition of the innovation likelihood:

$$\mathcal{L}\left(\mathbf{y}(t)\right) = \mathcal{N}\left(0; \mathbf{y}(t) - \mathbf{H}\mathbf{x}^f(t), \mathbf{H}\mathbf{P}^f(t)\mathbf{H}^\top + \mathbf{R}(t)\right)$$

with:

- $\mathbf{x}^f(t)$ the mean and $\mathbf{P}^f(t)$ the covariance of the forecast
- $\mathbf{P}^f(t)$ is depending on the model error covariance $\mathbf{Q}(t)$



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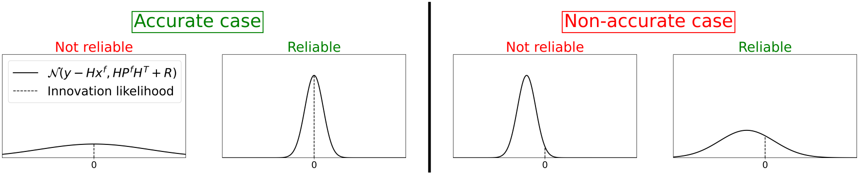
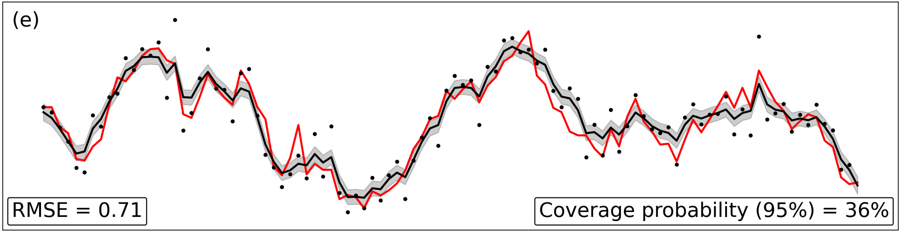
Graphical representation of the innovation likelihood:



$\Rightarrow$ **The innovation likelihood $\mathcal{L}\left(\mathbf{y}(\mathbf{t})\right)$ depends on $\mathbf{Q}(\mathbf{t})$ and $\mathbf{R}(\mathbf{t})$**

Confidence intervals and coverage probabilities

Kalman smoother with $Q = 0.1Q^t$ & $R = 0.1R^t$



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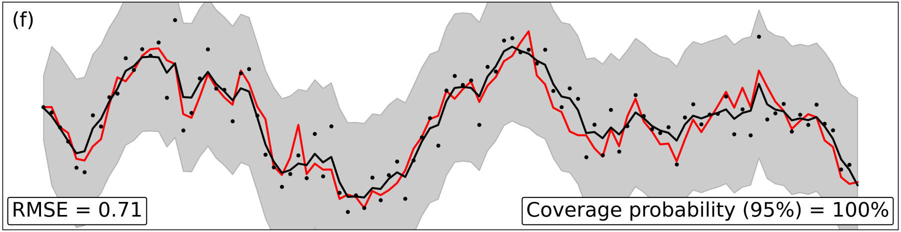

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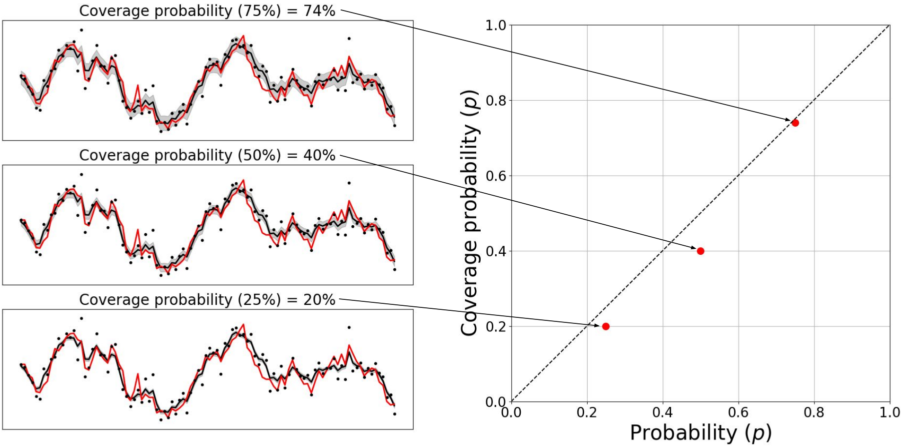

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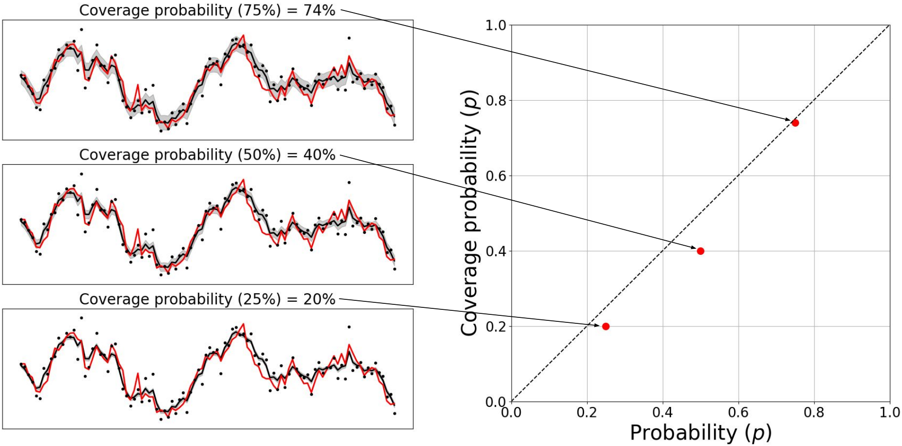
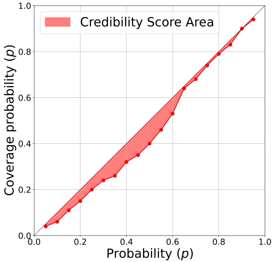


⇒ **The coverage probability should equal the probability of the corresponding confidence interval**

New metric for uncertainty quantification: credibility score[1]

Computation and visualization of the Credibility Score (CS):



Formula:

$$\text{CS} = \int_0^1 (\text{cov.prob.}(p) - p)\, dp$$

Property:

$$-0.5 \leq \text{CS} \leq 0.5$$

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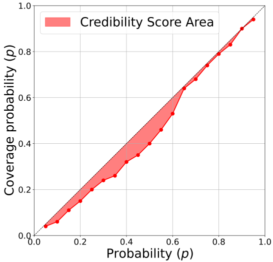

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Benefits of the credibility score:

- easy to compute (simple counting, variable by variable)
- interpretable ($CS < 0$ for overconfident, $CS > 0$ for underconfident)

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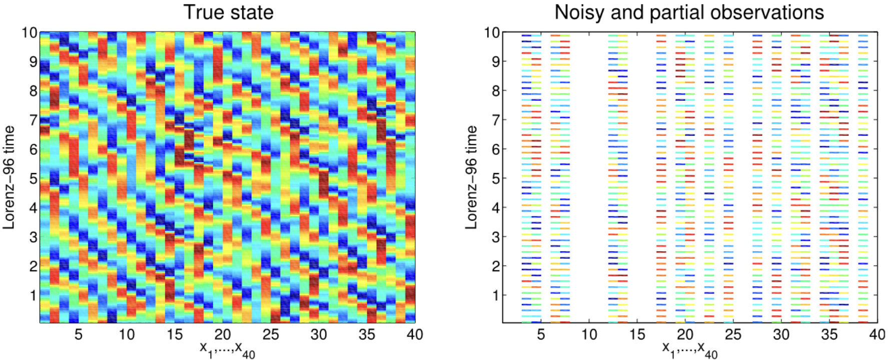
Application to the Lorenz-96 system

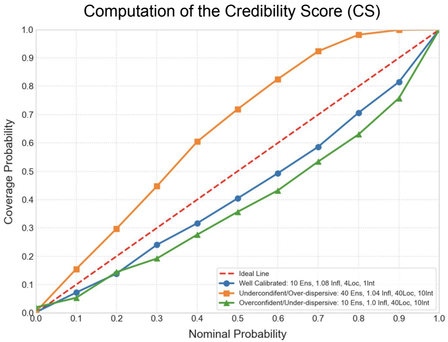
Lorenz-96 equations:

$$\frac{dx_k}{dt} = (x_{k+1} - x_{k-2})x_{k-1} - x_k + 8, \quad \forall k = 1, \ldots, 40$$



Application to the Lorenz-96 system

Experiment	#1	#2	#3
Ens. Memb.	10	40	10
Inflation	1.08	1.04	no
Localization	yes	no	no
RMSE	0.14	0.18	3.27
CRPS	2.04	2.14	2.16
CS	-0.07	0.14	-0.11



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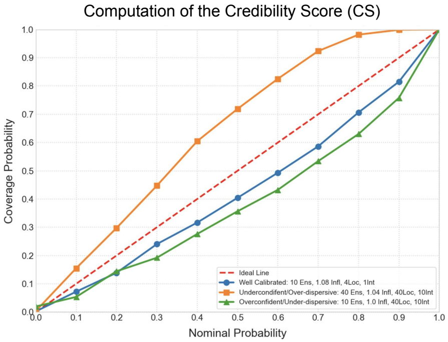

⇒ **CS provides information on accuracy and reliability**

⇒ **CS provides information on over- or under-confidency.**

Conclusions and perspectives

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Credibility score:

- new metric to quantify uncertainty (overconfidence, underconfidence)
- could be useful as loss function in ML and AI models

Merci pour votre attention !

Interested in this topic? Collaborations are welcome!

Contact:

- pierre.tandeo@imt-atlantique.fr
- www.tandeo.wordpress.com

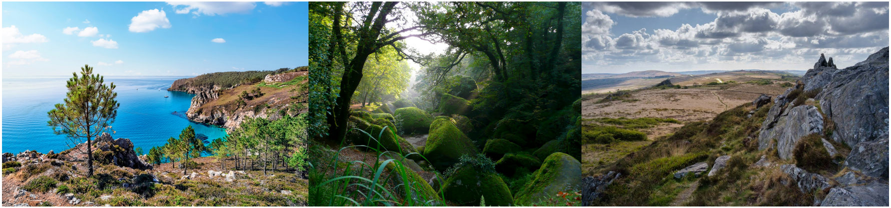

Figure: Nice places to visit near Brest, Bretagne, France

Inflation and localization of $\mathbf{P}^f$

Issues:

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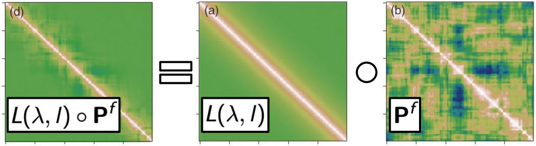

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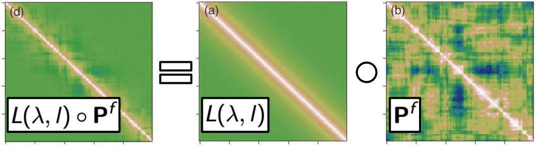


$\Rightarrow$ **The goal is to find, at each time t,** $\hat{\boldsymbol{\lambda}}(\mathbf{t}), \hat{\mathbf{l}}(\mathbf{t}) = \underset{\boldsymbol{\lambda}(\mathbf{t}), \mathbf{l}(\mathbf{t})}{\mathbf{argmax}} \left(\mathcal{L}(\mathbf{y}(\mathbf{t})) \right)$

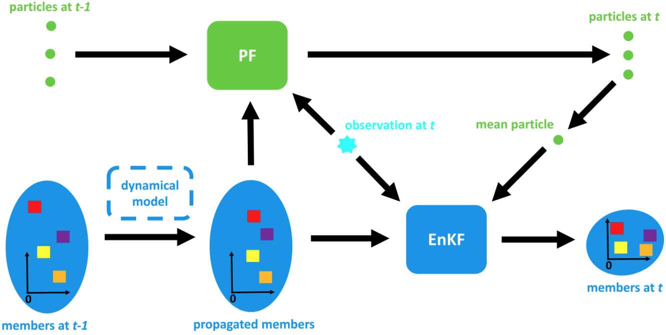
Combination of two data assimilation systems (PF-EnKF[1])

Ensemble Kalman Filter (EnKF):

- for the state ($\mathbf{x}$)
- provide an estimate of $\mathbf{P}^f(t)$

Particle Filter (PF):

- for the parameters (λ, l)
- weights prop. to $\mathcal{L}(\mathbf{y}(t))$



[1]Guillot et al. (2025). State and Stochastic Parameters Estimation with Combined Ensemble Kalman and Particle Filters. *Monthly Weather Review*

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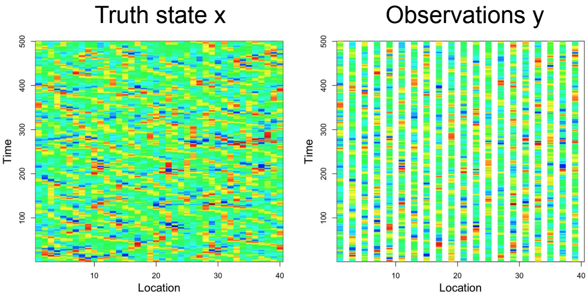

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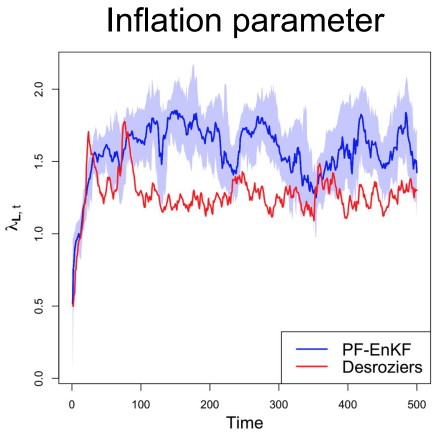
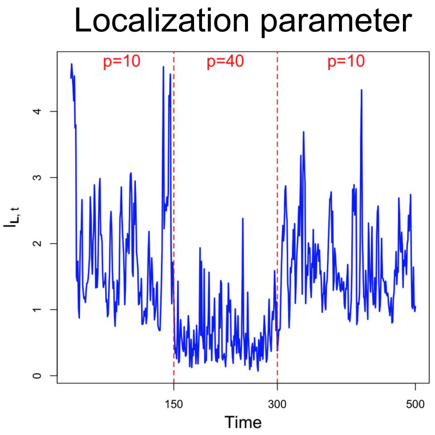
With:

- predefined **Q** and **R** matrices
- time-varying number of obs.

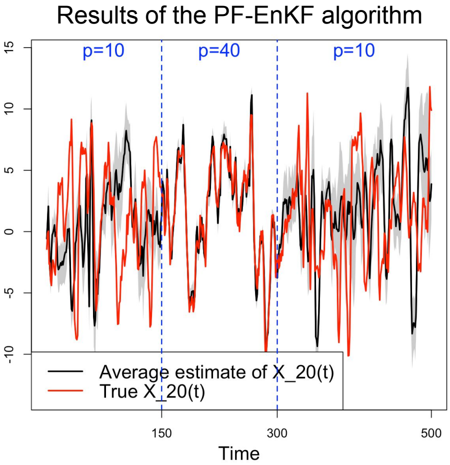
Estimation of inflation (λ) and localization (l) parameters





⇒ **PF-EnKF is able to estimate time-varying parameters**

Impact of PF-EnKF on the estimation of the state



<u>When $p = 40$ obs. variables:</u>

- good accuracy
- small confidence interval

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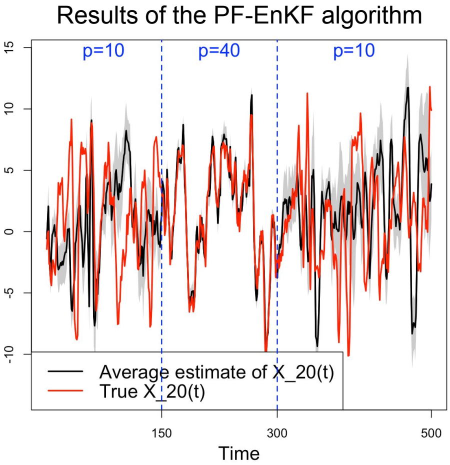

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Overall: Cov. prob. (70%) $\approx$ 70%

$\Rightarrow$ **Uncertainty is well calibrated (good coverage probability)**